9811 Introduction to AEA worksheet

Questions

© 2025 Pearson. All rights reserved.

1.

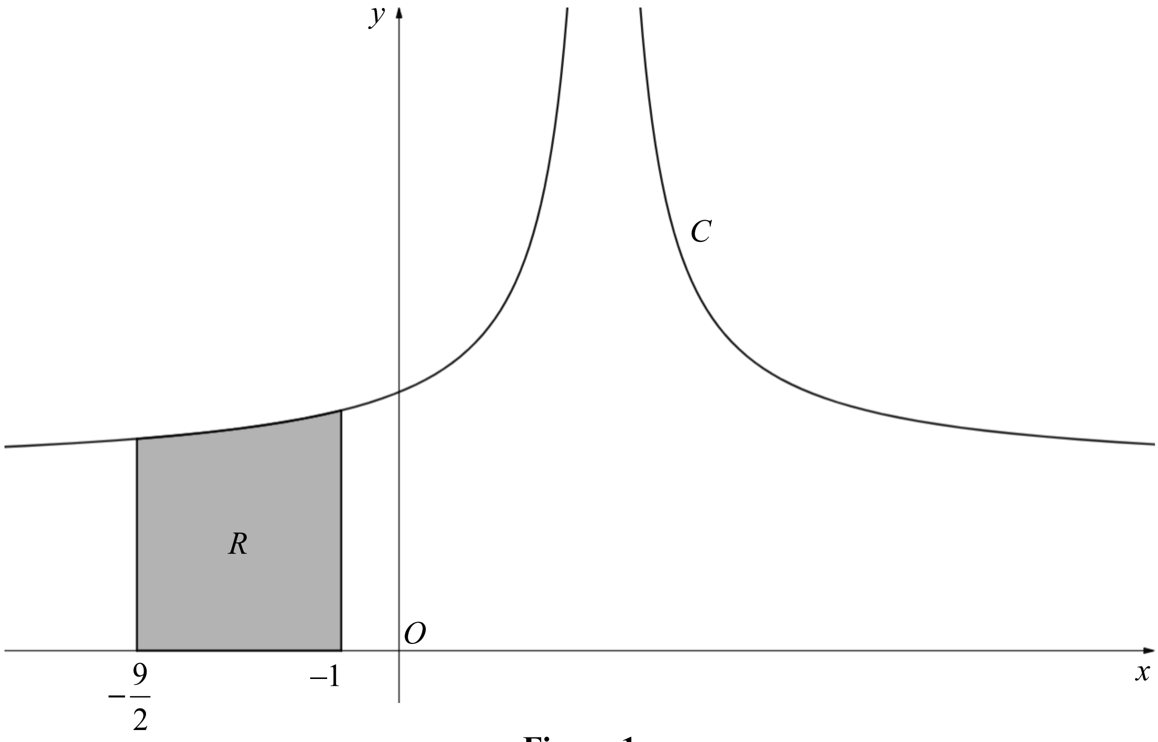

Figure 1

$$f(x) = \frac{10}{2x-7} \qquad x \in \mathbb{R} \quad x \neq k$$

$$g(x) = |x| + 3 \qquad x \in \mathbb{R}$$

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{gf}(x)$

(a) Determine the value of k

(1)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis, the line with equation $x = -1$, C and the line with equation $x = -\frac{9}{2}$.

(b) Show that the area of R is $a\ln\left(\frac{b}{c}\right) + d$ where a, b, c and d are constants to be determined.

(4)

(Total for Question 1 is 5 marks)

© 2025 Pearson. All rights reserved.

2.

$$f(x) = \sin x \qquad x \in \mathbb{R}$$
$$g(x) = e^x \qquad x \in \mathbb{R}$$

The curve with equation $y = f(x) \times gf(x) + a$ has the range $0 \leqslant y \leqslant b$ where a and b are constants.

(a) Determine the value of a and the value of b.

(3)

(b) Sketch, for $-\pi \leqslant x \leqslant 2\pi$, the curve with equation

$$y = f(x) \times gf(x) + a$$

Show on your sketch the coordinates of any points of intersection with the axes and the coordinates of any stationary points.

(2)

(Total for Question 2 is 5 marks)

© 2025 Pearson. All rights reserved.

3. (a) Show that the coefficient of x^r in the binomial expansion of $(1-x)^{-4}$ is

$$\frac{(r+1)(r+2)(r+3)}{6}$$

(3)

(b) Hence show that the coefficient of x^r in the expansion of

$$(3+2x-5x^2)(1-x)^{-4}$$

is

$$(r+1)(Ar+B)$$

where A and B are constants to be determined.

(4)

The infinite series S, is given by

$$S = 3 - 7 + \frac{33}{4} - \frac{15}{2} + \frac{95}{16} - \ldots$$

Given that S can be written as

$$S = \sum_{r=0}^{\infty}\left((r+1)(Ar+B)x^r\right)$$

where A and B were determined in part (b) and for some value of x,

(c) determine the value of S.

(3)

A student attempts to use the same approach to find the sum of the infinite series

$$3 - 28 + 132 - 480 + 1520 - \ldots$$

(d) Give one reason why the student will be unsuccessful in using this approach.

(1)

(Total for Question 3 is 10 marks)

© 2025 Pearson. All rights reserved.

4.

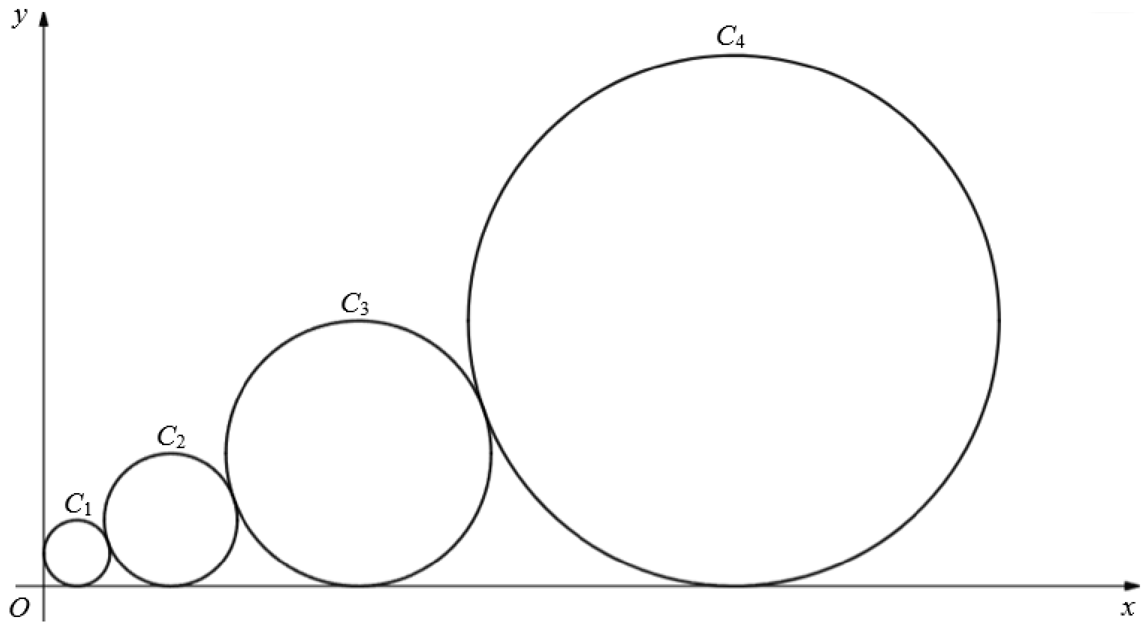

Figure 2

Figure 2 shows a sequence of circles, $C_1, C_2, C_3, \ldots$ which satisfy the following conditions:

- C_1 touches the positive y-axis
- C_n has centre (a_n, b_n) and radius r_n
- C_n touches the positive x-axis for all $n \in \mathbb{N}$
- C_n touches C_{n-1} once for all $n \in \mathbb{N}$
- $r_n = 2^n$ for all $n \in \mathbb{N}$
- a_n forms a strictly increasing sequence.

(a) Determine an equation for C_1

(2)

(b) Show that

$$a_n - a_{n-1} = 2\sqrt{b_n b_{n-1}}$$

(3)

(c) Show that

$$a_n = a_{n-1} + \mathrm{f}(n)\sqrt{2}$$

where $\mathrm{f}(n)$ is a fully simplified function to be determined.

(2)

© 2025 Pearson. All rights reserved.

(d) Prove that the centres of the circles lie on a straight line.

(3)

(e) Show that

$$a_n = 2 + 4\sqrt{2}\left(2^{n-1} - 1\right)$$

(3)

(+S1)

(Total for Question 4 is 14 marks)

© 2025 Pearson. All rights reserved.

5. A particle is projected from a point H metres above a horizontal plane.

The particle

- has initial speed $U\,\mathrm{m\,s^{-1}}$
- is projected an angle of elevation α above the horizontal, where α may vary
- moves freely under gravity until it strikes the horizontal plane

When the particle has travelled a horizontal distance x metres, its height above the horizontal plane is y metres.

(a) Show that

$$y = H + x\tan\alpha - \frac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$$

(5)

Given that the maximum horizontal distance travelled by the particle, R metres, is achieved when $\alpha = \beta$

(b) show that

$$R = \frac{U^2 \cot\beta}{g}$$

(5)

(c) Hence, show that

$$\beta = \arctan\left(\frac{U}{\sqrt{2gH+U^2}}\right)$$

(5)

(+S1)

(Total for Question 5 is 16 marks)

(TOTAL FOR QUESTION PAPER IS 50 MARKS)

© 2025 Pearson. All rights reserved.